Physics 431/531: Mechanics 1

Mid-Term Test 1

- You may use your textbook, notes, homework and homework solutions.
- Time: 1 hr. 15 minutes
- Answer three out of the four questions. Each is worth 10 points. (If you answer all four, best 3 scores will be used.)

Question 1

An object of mass $m$ that is constrained to move in one dimension is subject to a force that is dependent on both velocity $v$ and position $x$,

$$F(v, x) = -k xv,$$

where $k$ is a positive constant. The object has an initial velocity $v_0 > 0$ when it is at $x = 0$. Show that the object comes to a stop when

$$x = \sqrt{\frac{2mv_0}{k}}.$$

Hints: Recall that $\frac{dv}{dt} = v \frac{dv}{dx}$. Be a physicist, and treat $dv$ and $dx$ like regular variables. Remember to use the initial conditions given.

Question 2

The potential energy function for a particle of mass $m$ is

$$U(x, y, z) = Cxyz$$

where $C$ is a positive constant.

(a) What is the force on the particle at position $\mathbf{r}_0 = (x_0, y_0, z_0)$. [4 points]

(b) If the particle has speed $v_0$ at $(x_0, y_0, z_0)$, what is the speed of the particle if it as $(-x_0, -y_0, -z_0)$? [6 points]
**Question 3**

A loosely held pistol of mass $M$ with a single bullet of mass $m$ is held at a distance $h$ above the floor. The bullet is fired horizontally with an initial speed $v_b$, causing the pistol to recoil in the other direction. Assuming that the force with which the gun is held is negligible, what is the horizontal displacement of the pistol (the distance $d$ in the figure below) when it lands on the floor? Give your answer in terms of the variables given above and $g$, the acceleration due to gravity.

![Diagram of a pistol and bullet](image)

**Question 4**

A roller coaster of mass $m$ on a frictionless track starts with a speed of $v_0$. At the bottom of the track, the radius of curvature is $R$ (that is, the track is equivalent to a circular track of radius $R$). If the cart starts at a height $h$ above the bottom of the track, what is the magnitude of the normal force of the cart on the roller coaster at the bottom of the track? Give your answer in terms of $m$, $g$, $h$, $R$ and $v_0$.

![Diagram of a roller coaster track](image)