Spring 2006 Physics 262, Test 3: SOLUTIONS

Answer Key for Section A

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Section B

1. Water has a specific heat of $4 \times 10^3 \text{ J/(kg} \cdot \text{C}^\circ\text{)}$ and latent heat of fusion of $3.3 \times 10^5 \text{ J/kg}$. If I mix 2.0 kg of water at 20$^\circ$C with 1.0 kg of ice at 0$^\circ$C in a thermally insulated container, how much ice is left when the mixture comes to thermal equilibrium? (You may leave your answer in the form of a fraction.)

Answer:

Use conservation of energy: $Q_{\text{water}} + Q_{\text{ice}} = 0$, where $Q$ is positive when heat is absorbed. The final mixture will have ice and water at 0$^\circ$C. The heat absorbed by the water as it cools from 20$^\circ$C to 0$^\circ$C is

$$Q_{\text{water}} = m_{\text{water}} c_{\text{water}} \Delta T,$$

where $m_{\text{water}} = 2.0 \text{ kg}$, $c_{\text{water}} = 4 \times 10^3 \text{ J/(kg} \cdot \text{C}^\circ\text{)}$, and $\Delta T = 0^\circ\text{C} - 20^\circ\text{C} = -20^\circ\text{C}$. Therefore,

$$Q_{\text{water}} = (2.0 \text{ kg}) \cdot (4 \times 10^3 \text{ J/(kg} \cdot \text{C}^\circ\text{)}) \cdot (-20^\circ\text{C}) = -1.6 \times 10^5 \text{ J}.$$

From conservation of energy,

$$Q_{\text{ice}} = -Q_{\text{water}} = 1.6 \times 10^5 \text{ J}.$$

This melts mass $m_{\text{melt}}$ of the ice, given by

$$m_{\text{melt}} L = Q_{\text{ice}}$$

where $L = 3.3 \times 10^5 \text{ J/kg}$ is the latent heat of fusion. Solving for $m_{\text{melt}}$ gives

$$m_{\text{melt}} = \frac{Q_{\text{ice}}}{L} = \frac{1.6 \times 10^5 \text{ J}}{3.3 \times 10^5 \text{ J/kg}} = \frac{16}{33} \text{ kg}.$$

Therefore, the mass of the ice that remains is $1.0 \text{ kg} - \frac{16}{33} \text{ kg} = \frac{17}{33} \text{ kg}$. 
2. For which of the following transitions in a hydrogen atom is the wavelength of the emitted light the longest? (1) \( n = 2 \) to \( n = 1 \) (2) \( n = 3 \) to \( n = 1 \) (3) \( n = \infty \) to \( n = 2 \). Justify your answer quantitatively (that is, with numbers).

Answer:

The energy of the photon \( E_{\text{photon}} \) is given by conservation of energy, \( E_{\text{photon}} + \Delta E = 0 \Rightarrow E_{\text{photon}} = -\Delta E \), where \( \Delta E \) is the change in energy of the hydrogen atom. The energy levels of the hydrogen are \( E_n = -\frac{E_R}{n^2} \), where \( E_R = 13.6 \text{ eV} \) and \( n \) are integers.

(1) For \( n = 2 \to n = 1 \), \( \Delta E = -E_R \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = -\frac{3}{4} E_R \Rightarrow E_{\text{photon}} = \frac{3}{4} E_R \).

(2) For \( n = 3 \to n = 1 \), \( \Delta E = -E_R \left( \frac{1}{1^2} - \frac{1}{3^2} \right) = -\frac{8}{9} E_R \Rightarrow E_{\text{photon}} = \frac{8}{9} E_R \).

(3) For \( n = \infty \to n = 2 \), \( \Delta E = -E_R \left( \frac{1}{\infty^2} - \frac{1}{2^2} \right) = -\frac{1}{4} E_R \Rightarrow E_{\text{photon}} = \frac{1}{4} E_R \).

Since \( E_{\text{photon}} = hf = hc/\lambda \), the photon with the lowest energy has the longest wavelength. Since \( \frac{1}{4} < \frac{3}{4} < \frac{8}{9} \), the lowest energy and longest wavelength photon corresponds to (3) \( n = \infty \) to \( n = 2 \) transition.