Spring 2006 Physics262, Test 3: SOLUTIONS

Answer Key for Section A

1. D	5. A	9. C	13. A	17. B	21. A
2. C	6. D	10. A	14. A	18. D	
3. C	7. C	11. A	15. B	19. A	
4. D	8. A	12. B	16. D	20. B	

Section B

1. Water has a specific heat of $4 \times 10^3\,\mathrm{J/(kg \cdot C^o)}$ and latent heat of fusion of $3.3 \times 10^5\,\mathrm{J/kg}$. If I mix 2.0 kg of water at 20 °C with 1.0 kg of ice at 0 °C in a thermally insulated container, how much ice is left when the mixture comes to thermal equilibrium? (You may leave your answer in the form of a fraction.)

   Answer:

   Use conservation of energy: $Q_{\mathrm{water}} + Q_{\mathrm{ice}} = 0$, where Q is positive when heat is absorbed. The final mixture will have ice and water at 0 °C. The heat absorbed by the water as it cools from 20°C to 0°C is

   $$Q_{\mathrm{water}} = m_{\mathrm{water}}\, c_{\mathrm{water}}\, \Delta T,$$

   where $m_{\mathrm{water}} = 2.0\,\mathrm{kg}$, $c_{\mathrm{water}} = 4 \times 10^3\,\mathrm{J/(kg \cdot C^o)}$, and $\Delta T = 0\,{}^\circ\mathrm{C} - 20\,{}^\circ\mathrm{C} = -20\,{}^\circ\mathrm{C}$. Therefore,

   $$Q_{\mathrm{water}} = (2.0\,\mathrm{kg}) \cdot (4 \times 10^3\,\mathrm{J/(kg \cdot C^o)}) \cdot (-20\,{}^\circ\mathrm{C}) = -1.6 \times 10^5\,\mathrm{J}.$$

   From conservation of energy,

   $$Q_{\mathrm{ice}} = -Q_{\mathrm{water}} = 1.6 \times 10^5\,\mathrm{J}.$$

   This melts mass m_{melt} of the ice, given by

   $$m_{\mathrm{melt}} L = Q_{\mathrm{ice}}$$

   where $L = 3.3 \times 10^5\,\mathrm{J/kg}$ is the latent heat of fusion. Solving for m_{melt} gives

   $$m_{\mathrm{melt}} = \frac{Q_{\mathrm{ice}}}{L} = \frac{1.6 \times 10^5\,\mathrm{J}}{3.3 \times 10^5\,\mathrm{J/kg}} = \frac{16}{33}\,\mathrm{kg}.$$

   Therefore, the mass of the ice that remains is $1.0\,\mathrm{kg} - \frac{16}{33}\,\mathrm{kg} = \boxed{\frac{17}{33}\,\mathrm{kg}.}$

2. For which of the following transitions in a hydrogen atom is the wavelength of the emitted light the *longest*? (1) $n = 2$ to $n = 1$ (2) $n = 3$ to $n = 1$ (3) $n = \infty$ to $n = 2$. Justify your answer quantitatively (that is, with numbers).

Answer:

The energy of the photon E_{photon} is given by conservation of energy, $E_{\text{photon}} + \Delta E = 0 \Rightarrow E_{\text{photon}} = -\Delta E$, where ΔE is the change in energy of the hydrogen atom. The energy levels of the hydrogen at are $E_n = -E_R/n^2$, where $E_R = 13.6\,\text{eV}$ and n are integers.

(1) For $n = 2 \rightarrow n = 1$, $\Delta E = -E_R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = -\frac{3}{4}E_R \Rightarrow E_{\text{photon}} = \frac{3}{4}E_R$.

(2) For $n = 3 \rightarrow n = 1$, $\Delta E = -E_R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = -\frac{8}{9}E_R \Rightarrow E_{\text{photon}} = \frac{8}{9}E_R$.

(3) For $n = \infty \rightarrow n = 2$, $\Delta E = -E_R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = -\frac{1}{4}E_R \Rightarrow E_{\text{photon}} = \frac{1}{4}E_R$.

Since $E_{\text{photon}} = hf = hc/\lambda$, the photon with the lowest energy has the longest wavelength. Since $\frac{1}{4} < \frac{3}{4} < \frac{8}{9}$, the lowest energy and longest wavelength photon corresponds to (3) $n = \infty$ to $n = 2$ transition.